Equally Likely Outcomes

Elementary Examples

Example 1: Suppose a single coin is flipped once. What is the probability that it lands face up (Heads)?

Example 2: Suppose a single die is rolled once. What is the probability that it lands with the "3" face up?

Equally Likely Outcomes

Slightly More Complicated Examples

Example 3: Suppose a quarter and a dime are each flipped once. What are the possible outcomes of the flips and what is the chance of getting two Heads? What is the chance of getting one head and one tail?

Example 4: Suppose a penny, nickel, dime and quarter are each flipped once. Determine the possible outcomes of the flips and the probabilities of each outcome.

Equally Likely Outcomes

Slightly More Complicated Examples

Example 5: Suppose two dice are rolled. Determine the possible outcomes of the roll and the probability that the sum of the dice is 6.

Equally Likely Outcomes

Determining Probabilities

- Identify how many possible outcomes there are (N).
- Identify the outcomes of interest (A) and determine the number of these (k).
- The probability is given by the ratio
$$P(A) = \frac{k}{N} \; .$$

Example 6: Suppose three coins are flipped. What is the probability that exactly two of the coins show heads?

Equally Likely Outcomes

Challenging Examples

Example 7: The game of Blackjack starts with the player receiving 2 cards, one face down and one face up. The player automatically wins if he or she is dealt a "Blackjack", a jack and an ace. What is the chance of being dealt a Blackjack?

Equally Likely Outcomes

Challenging Examples

Example 8: One game of poker consists of dealing five cards to each player. The hands are then ranked according to the types of cards:

Royal Flush	AKQJ10 of the same suit
Straight Flush	5 consecutive cards of the same suit
Full House	a pair and three of a kind
Four-of-a-kind	4 cards of the same denomination
Flush	5 cards of the same suit
Straight	5 consecutive cards (any suits)
Three-of-a-Kind	
Two Pairs	
Pair	
High card	

What are the chances of being dealt a Royal Flush? a Full House?

Equally Likely Outcomes

Counting Rules

Theorem 2.1: With m elements $a_1, a_2, \ldots, a_m$ and n elements $b_1, b_2, \ldots, b_n$ it is possible to form mn pairs containing one element from each group.

Definition 2.7: An *ordered* arrangement of r distinct objects is called a *permutation.*

The number of ways of ordering n distinct objects taken r at a time is

$$P_r^n = n(n-1)\cdots(n-r+1).$$

Definition: Factorials are defined to be

$$0! = 1, \qquad n! = n(n-1)(n-2)\cdot 2 \cdot 1.$$

Equally Likely Outcomes

Counting Rules

Definition 2.8: The number of *combinations* of n objects taken r at a time is the number of subsets, each of size r, that can be formed from the n objects.

The number of combinations is

$$C_r^n = \binom{n}{r} = \frac{n!}{r!(n-r)!}\ .$$

Equally Likely Outcomes

Example 8 revisited: Determine the probability of getting a hand having Three-of-a-Kind.

Equally Likely Outcomes

Counting Rules

Theorem 2.3: The number of ways of partitioning n distinct objects into k distinct groups containing $n_1, n_2, \ldots, n_k$ objects, respectively, is

$$\frac{n!}{n_1!n_2!\cdots n_k!}$$

where $n = \Sigma_{i=1}^{k} n_i$.

Equally Likely Outcomes

Example 2.10: A labor dispute has arisen concerning the alleged distribution of twenty laborers to four different construction jobs. The first job (considered to be abominable employment) required six laborers; the second, third and fourth utilized four, five and five laborers, respectively.

The dispute arose over an alleged random distribution of the laborers to the jobs which placed all four members of a particular ethnic group on job 1. In considering whether the assignment represented injustice, a mediation panel desired the probability of the observed event.

Equally Likely Outcomes

Example: Evidence of Intelligent Life

Discovery Toys manufactures a set of plastic links of varying colors. One particular set consists of 2 red links, 4 orange links, 4 yellow links, 4 green links , 4 blue links and 3 purple links. A toddler is able to snap the links together.

Suppose a toddler forms a chain in which all the links of the same color are together. What are the chances such an event would happen if the toddler could not distinguish the colors and so randomly chose links to attach?