1 Mathematical Induction

Notation 1.1

$$\mathbb{N} = \{1, 2, 3, \ldots\} \text{ — the set of } \textit{natural} \text{ numbers.}$$

$$\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\} \text{ — the set of } \textit{integers}.$$

Definition 1.1 *Mathematical Induction* Let $A_1, A_2, A_3, \ldots, A_n, \ldots$ be a sequence of statements, each of which can be either true or false. Suppose, we know that A_1 is true. Also, suppose that for each natural number k we know that "A_k is true" implies "A_{k+1} is true" (i.e. each one implies the next one). Then all statements $A_1, A_2, A_3, \ldots, A_n, \ldots$ are true.

Problem 1.1 Prove by induction:

$$\sum_{i=1}^{n}(2i-1) = n^2$$

Problem 1.2 Prove by induction:

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

Problem 1.3 Prove by induction:

$$\sum_{i=1}^{n} i^3 = (\sum_{i=1}^{n} i)^2$$

Problem 1.4 Find

$$\sum_{k=1}^{n} k * k!$$

Problem 1.5 Prove *Bernoulli inequality*: if $a > 0$ then $(1+a)^n \geq 1 + na$ for all $n \in \mathbb{N}$.

Problem 1.6 Prove by induction: $2n + 1 \leq n^2$ for $n \geq 3$.

Problem 1.7 Prove by induction: $2^n \geq n^2$ for $n \geq 4$.

Problem 1.8 In how many regions do n lines in general position divide a plane? (The lines are *in general position* if no two lines are parallel, and no three lines pass through the same point)