

6.2 HYPOTHESIS TESTING- EXAMPLES

Stating the Null and Alternative

1. An ecologist would like to show that Milwaukee has an air pollution problem. Specifically, she would like to show that the mean level of carbon monoxide in downtown Milwaukee air is higher than 4.9 parts per million. State the null and the alternative hypotheses.

Solution: To state the hypotheses we first need to identify the population parameter in question and the value to which it is being compared. The “mean level of carbon monoxide pollution” is the mean level μ and the 4.9 parts per million is the specific value. Our ecologist is questioning the value of μ .

Recall:

- i. The null hypothesis states that the parameter in question has a specified value.
- ii. A sample mean is going to be used as the basis for an inference about the population mean, and sample means have an approximately normal distribution as described by the central limit theorem.
- iii. A normal distribution is determined when its mean and standard deviation are specified.

All this is suggesting that the statement containing the equal sign will become the null hypothesis; the other statement becomes the alternative hypothesis. Thus we have:

$$H_0 : \mu = 4.9$$

$$H_a : \mu > 4.9$$

Recall that once the null hypothesis is stated, we proceed with the hypothesis test under the assumption that the null hypothesis is true. Thus $\mu = 4.9$ locates the center of the sampling distribution of sample means. For that reason the null hypothesis will be written with an equal sign only.

2. The “mean level of carbon monoxide in downtown Milwaukee is not 4.9 parts per million”. State the null and alternative hypotheses that correspond to this statement.

Solution:

$$H_0 : \mu = 4.9$$

$$H_a : \mu \neq 4.9$$

The Complete Hypothesis Testing Procedure

3. For many semesters an instructor has recorded his students' grades, and the mean for all these students' grades, μ , is 72. The current class of 36 students seems to be better than average in ability and the instructor wants to show that according to their average "the current class is superior to the previous classes". Does the class mean of 75.2 present sufficient evidence to support the instructor's claim that the current class is superior? Use $\alpha = 0.005$ and $\sigma = 12$

Solution:

To be superior this class must have a mean grade that is higher than the mean of all previous classes.

Step 1: $H_0 : \mu = 72$

Step 2: $H_a : \mu > 72$ (class is superior)

Step 3: The level of significance $\alpha = 0.05$ is given in the statement of the problem. The standard score z is used as the test statistic when the null hypothesis is about a population mean and the standard deviation is known. The critical region –i.e. values of the standard score z that will cause a rejection of the null hypothesis- has an area of 0.05 and is located at the extreme right of the distribution. The critical region is on the right because large values of the sample mean suggest "superior", while values near or below 72 support the null hypothesis. The critical region has an area of 0.05 and a critical value of +1.65 (obtained from z table).

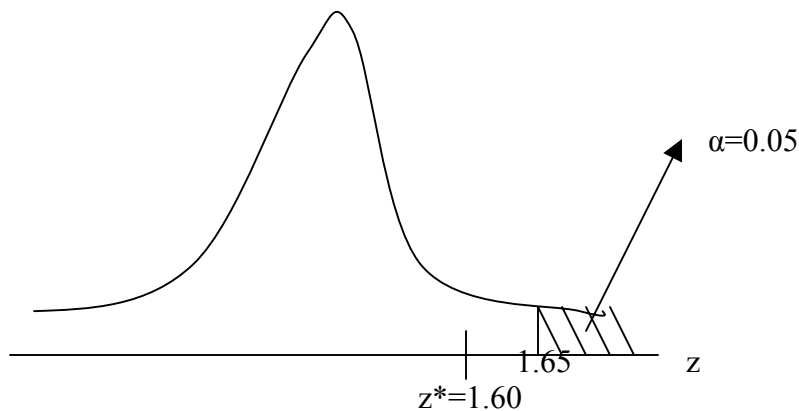
Step 4: The value of the test statistic z will be found by using formula $Z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$

And the sample information $\bar{x} = 75.2$ and $n = 36$.

$$Z = \frac{75.2 - 72}{12 / \sqrt{36}} = \frac{3.2}{2} = 1.6$$

$Z^* = 1.60$ (we will use an asterisk, *, for the calculated value of the test statistic)

Step 5: We now compare the calculated test statistic, z^* , to the test criteria set up in Step 3 by locating the calculated value on the diagram and placing an asterisk at that value.



Since the test statistic (calculated value) falls in the acceptance region (unshaded portion of the diagram), we must reach the following decision:

Decision: Fail to reject H_0 .

Recall that the critical region was to be shaded in the diagram, and when the test statistic falls in the critical region, we must reject H_0 .

Step 5 is then completed by stating a conclusion:

Conclusion: There is not sufficient evidence to show that the current class is superior.

EXERCISES

1. It has been claimed that the mean weight of women students at UWM is 54.4 kilograms. Statistics instructor Eren does not believe the statement that the mean is 54.4 kilograms. To test the claim she collects a random sample of 100 weights from among the women students. A sample mean of 53.75 kilograms results. Is this sufficient evidence to reject the null hypothesis? Use $\alpha = 0.05$ and $\sigma = 5.4$ kilograms.
2. The student body at many colleges is considered to be a “commuter population”. Because of this, the following question was asked of the Student Affairs Office: “How far (one way) does the average college student commute to college daily?” The office answered: “No more than 9 miles.” The inquirer was not convinced of the truth of this and decided to test the statement. A sample of 50 students was taken and a mean commuting distance of 10.22 miles was found. Test the hypothesis stated above at a significance level of $\alpha = 0.05$, using $\sigma = 5$ miles.

University of Wisconsin Milwaukee
ECON 210-002 Spring 2004
Ozlem Eren