

**MATH 413**  
**Fall 2007**  
**Lab12**

**Exercise 1**

(5 pt)

The linear system  $Ax = b$  given by

$$\begin{bmatrix} 1 & 2 \\ 1.00001 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 3.00001 \end{bmatrix}$$

has solution  $(1, 1)^t$ .

Use five-digit rounding arithmetic to find the solution of the perturbed system

$$\begin{bmatrix} 1 & 2 \\ 1.000011 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3.00001 \\ 3.00003 \end{bmatrix}$$

and compare the change in  $A$  and  $b$  to the change in  $x$ .

Also calculate the condition number of the matrix  $A$ .

**Exercise 2**

(5 pt)

Calculate the condition number of the Hilbertmatrix

$$A = \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} \\ \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} \end{bmatrix}$$

and solve the system  $Ax = b$  for

$$b = \begin{bmatrix} \frac{25}{12} \\ \frac{77}{19} \\ \frac{30}{420} \end{bmatrix}$$