Review Homework

Math 116

Exercises 1-4 due Monday
Exercises 5-7 due Wednesday

Exercise 1 **(15 pt)**
Let $f(x) = x - 5$ and $g(x) = \sqrt{2x+3}$. Find $(g \circ f)(x)$ and its domain.

$(g \circ f)(x) = g(x-5) = \sqrt{2x-7}$. Now determine the domain of this new function. Notice that f has no restrictions on its domain. So we only need to look at the domain of $h(x) = \sqrt{2x-7}$. The domain of this function is $[\frac{7}{2}, \infty)$. Hence this is the domain of $(g \circ f)(x)$.

Exercise 2 **(15 pt)**
Given the function $f(x) = \sqrt{32-x}$ is one-to-one on the interval $(-\infty, 32)$, find its inverse. What is the domain and range of $f^{-1}(x)$?

First of all, note that the range of f^{-1} is always equal to the domain of $f(x)$. So range of $f^{-1}(x)$ is $(-\infty, 32)$. The range of $f(x)$ is $(0, \infty)$ which is equal to the domain of $f^{-1}(x)$.

$$\begin{aligned} y &= \sqrt{32-x} \\ x &= \sqrt{32-y} \\ x^2 &= 32-y \\ f^{-1}(x) &= 32-x^2 \end{aligned}$$

Exercise 3 **(10 pt)**

Consider the function $f(x) = x^3 - x^2 + 25x - 25$.
Find all three solutions of $f(x) = 0$. What are the x-intercepts?
Use the rational roots theorem to list all possible rational roots of $f(x)$:
$\pm 1, \pm 5, \pm 25$
We find that 1 is a solution of $f(x) = 0$. We are left with $x^2 + 25$ when we divide $f(x)$ by the factor $(x - 1)$.

$$\begin{aligned} x^2 + 25 &= 0 \\ x^2 &= -25 \\ x &= \pm 5i \end{aligned}$$

So all solutions are : x= $1, \pm 5i$
x-intercept is only x=1 (real zeros)

Exercise 4 **(15 pt)**

Solve

$$\frac{13x - 10}{x^2 + 6x} \leq 1$$

$$\begin{aligned} \frac{13x - 10}{x^2 + 6x} &\leq 1 \\ \frac{13x - 10}{x^2 + 6x} - 1 &\leq 0 \\ \frac{13x - 10 - x^2 - 6x}{x^2 + 6x} &\leq 0 \\ \frac{-x^2 + 7x - 10}{x(x+6)} &\leq 0 \\ \frac{-(x^2 - 7x + 10)}{x(x+6)} &\leq 0 \\ \frac{(x^2 - 7x + 10)}{x(x+6)} &\geq 0 \\ \frac{(x-5)(x-2)}{x(x+6)} &\geq 0 \end{aligned}$$

If we divide by a negative number, we have to change the sign!
Critical numbers are: 0, -6 (denominator), 5, 2 (numerator)

(a,b)	testnumber	sign
$(-\infty, -6)$	-7	+
$(-6, 0)$	-1	-
$(0, 2)$	1	+
$(2, 5)$	3	-
$(5, \infty)$	6	+

Hence: $(-\infty, -6) \cup (0, 2] \cup [5, \infty)$
Since we have $\leq$ endpoints are included, but only the endpoints which are zeros of the numerator!!!
-6 and 0 are excluded from the domain of the inequality, so cannot be included in the solutions set.