Review Homework Solution 1st part

Math 116

Exercises 1-4

1. (10 points) Solve for x: $x^2 - 4x + 7 = 0$. Simplify your answer as much as possible. Check by substitution.

   $x = \frac{4 \pm \sqrt{16-4*7}}{2} = \frac{4 \pm \sqrt{-12}}{2} = \frac{4 \pm 2i\sqrt{3}}{2} = 2 \pm i\sqrt{3}$
   CHECK:
   $(2 \pm i\sqrt{3})^2 - 4(2 \pm i\sqrt{3}) + 7 = 0$
   $4 \pm 4i\sqrt{3} + 3i^2 - 8 \mp 4i\sqrt{3} + 7 = 0$
   $4 - 3 - 8 + 7 = 0$
   $0 = 0$
   Our answer checks, so we can be confident it is correct.

2. (20 points) Give the quotient and remainder if $x^4 + x^3 - 7x^2 + 8x + 1$ is divided by $x^2 + 3x - 2$. Check your answer by remultiplying and adding the remainder.

   If you divide you get $x^2 - 2x + 1, R(x+3)$

   CHECK:

$$\begin{aligned}(x^2+3x-2)(x^2-2x+1)+(x+3) &= (x^4+3x^3-2x^2-2x^3-6x^2+4x+x^2+3x-2)+(x+3)\\ &= (x^4+x^3-7x^2+7x-2)+(x+3)\\ &= x^4+x^3-7x^2+8x+1\end{aligned}$$

   Our answer checks, so we can be confident it is correct.

3. (15 points) Carefully graph the lines $3x + 2y = 8$ and $6x - 4y = 3$ below. Find the exact coordinates of their points of intersection algebraically, and check your answer by substitution in the given equations.

$$\begin{aligned}6x+4y &= 16\\ 6x-4y &= 3\\ 12x &= 19\\ x &= \frac{19}{12}\end{aligned}$$

$$\begin{aligned}
3\left(\frac{19}{12}\right) + 2y &= 8 \\
\frac{19}{4} + 2y &= 8 \\
2y &= \frac{32}{4} - \frac{19}{4} \\
2y &= \frac{13}{4} \\
y &= \frac{13}{8}
\end{aligned}$$

To check this answer, we plug the solution into the first equation:

$$\begin{aligned}
3 \cdot \frac{19}{12} + 2 \cdot \frac{13}{8} &= \frac{19}{4} + \frac{13}{4} \\
&= \frac{32}{4} \\
&= 8
\end{aligned}$$

and into the second equation:

$$\begin{aligned}
6 \cdot \frac{19}{12} - 4 \cdot \frac{13}{8} &= \frac{19}{2} - \frac{13}{2} \\
&= \frac{6}{2} \\
&= 3
\end{aligned}$$

So the final solution is $(\frac{19}{12}, \frac{13}{8})$.

4. (10 points) What is the largest set of real numbers on which the function whose rule is $f(x) = \frac{2x+5}{x+1}$ is defined. Solve the equation $f(x) = y$ for x.

   For the domain of $f(x)$ to be defined, we must make sure the denominator does not equal zero. This only happens when $x = -1$, so our domain is $\{x \text{real} | x \neq -1\}$. Now to solve for x in $f(x) = y$:

$$\begin{aligned}
y &= \frac{2x+5}{x+1} \\
y(x+1) &= 2x+5 \\
yx + y &= 2x+5 \\
y - 5 &= 2x - yx \\
y - 5 &= x(2-y) \\
\frac{y-5}{2-y} &= x
\end{aligned}$$