Asymptotic Properties of the MLE of Parameters of the Multivariate O-U Process

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Abstract

Computer modeling is having a profound effect on scientific research, replacing direct physical experimentation by computer simulation of complex models. A computer experiment is a run of computer codes with various inputs. However, in most cases running the computer codes for all inputs is impossible. One approach to generate a measure of uncertainty at any input is to treat the computer output as a realization of a stochastic process, $X(t)$. Usually $X(t)$ is used to model the departure from an assumed linear model.

In this research $X(t)$ is assumed to be a multivariate, three-dimensional (time) Ornstein-Uhlenbeck (O-U) process with covariance function $V(\sigma^2, \theta, t, s) = \sigma^2 D e^{-\sum_{i=1}^{3} \theta_i |t_i - s_i|}$ where $t' = (t_1, t_2, t_3)$ and $s' = (s_1, s_2, s_3) \in T$, $\sigma^2 > 0$, $\theta' = (\theta_1, \theta_2, \theta_3) \in [a, b]^3 \subset (0, \infty)^3$, and D is a known positive definite p-dimensional matrix. It is shown that the maximum likelihood estimators $\hat{\sigma}^2$ and $\hat{\theta}_i$, $i = 1, \ldots, 3$ are strongly consistent and asymptotically normal when the observations are taken from a complete lattice, not necessarily equally spaced.