

## Additional two point problems for assignment 5

1. Suppose that an  $n \times n$  matrix  $M$  is a matrix of the form

$$M = \begin{bmatrix} A & 0 \\ B & C \end{bmatrix}$$

$A$  is a  $p \times p$  matrix,  $C$  is a  $q \times q$  matrix with  $p + q = n$ , and  $0$  is the  $p \times q$  matrix of all 0's. Prove that

$$M^k = \begin{bmatrix} A^k & 0 \\ B_k & C^k \end{bmatrix}$$

where

$$B_k = BA^{k-1} + CBA^{k-2} + \dots + C^j BA^{k-j-1} + \dots + C^{k-2} BA + C^{k-1} B.$$

2. Suppose that

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1/3 & 1/6 & 1/2 & 0 \\ 1/10 & 1/5 & 3/10 & 2/5 \end{bmatrix}$$

Use the result of the preceding exercise to compute the limit as  $k \rightarrow \infty$  of  $P^k$  and interpret each of the entries of the limit in terms of an appropriate Markov chain. Use a computing device to verify your answers numerically.