Additional two point problems for assignment 4

1. Solve the following difference equation: $f_{n+3} = 4f_{n+2} - 2f_{n+1} - f_n + n^2$, $f_0 = f_1 = f_2 = 1$.

2. Gambler's ruin with ties. Suppose that each player tosses his own coin. Player 1 has a coin that comes up heads with probability p and player two has a coin that comes up heads with probability q. If the coins do not match, the player with the head wins a dollar. If the coins match, no money changes hands. If the total capital in the game is N, find a formula for the probability that player 1 is ruined if that player starts with x dollars.

3. Gambler's ruin as a single player game with higher stakes. The player is armed with a spinner which comes up -2 with probability 1/2, 1 with probability 1/3, and 2 with probability 1/6. If the spinner comes up x, the gambler's fortune changes by x. If he has only one dollar and the spinner comes up -2 the game is over. The same is true if the spinner comes up 1 or 2 and the gambler's fortune is N-1. What is the probability that the gambler is ruined if his intial stake is $y \in \{0, \cdots, N\}$?

4. Suppose that an $N \times N$ matrix M has the form $M_{i,j} = 0$ if $i \in \{2, \ldots, N\}$ and $j \neq i-1$, and $M_{i,i-1} = 1$. Show that each eigenvalue of M has a one-dimensional eigenspace that is spanned by a vector whose entries are descending powers of the eigenvalue. Use this result to find the eigenvectors of the matrix

$$\begin{bmatrix} 4 & 3 & -6 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$