Lindeberg's Theorem: The Central Limit Theorem for Unequal Components

Theorem 1 (Lindeberg's Theorem) *Suppose that $X_1, X_2, \ldots,$ are independent random variables with*

$$\begin{aligned} \mathrm{E}[X_k] &= 0 \\ \mathrm{E}[X_k^2] &= v_k \end{aligned}$$

Put $V_n = v_1 + \ldots + v_n$, and assume that for every $\epsilon > 0$ there exists $N > 0$ such that if $n \geq N$ then

$$\frac{\max_{k=1,\ldots,n}\{v_k\}}{V_n} \leq \epsilon.$$

Then

$$\lim_{n\to\infty} \Pr\left(\frac{X_1 + \cdots + X_n}{\sqrt{V_n}} \leq a\right) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{a} \exp\left(\frac{-u^2}{2}\right)\, du$$