Prerequisite review items

Here is what you are expected to know already about linear algebra.

1. The algebra of matrices:
   (a) How to add and subtract matrices.
   (b) How to multiply matrices.
   (c) What an identity matrix is.
   (d) How to find the inverse of a matrix
2. The relation between systems of equations and matrices.
3. How to compute a determinant of a matrix and the relation between determinants and the existence of matrix inverses:

   A square matrix A is invertible if and only if the determinant of A is not zero.
4. How to compute the characteristic polynomial of a square matrix.
5. The definition of an eigenvalue/eigenvector pair of square matrix A:

   If α is a non-zero vector and x is scalar, then (α, x) is an eigenvector/eigenvalue pair of the square matrix A if $A\alpha = x\alpha$.
6. The definition of vector space.
7. The definition of linear transformation:

   If V and W are vector spaces and $T : V \to W$, we say that T is a linear transformation if $T(\alpha + x\beta) = T(\alpha) + xT(\beta)$ for ever pair of vectors α and β in V and every scalar x.
8. The definition of linear independence of vectors:

   If V is a vector space and $S = \{\alpha_1, \ldots, \alpha_N\} \subset V$, we say that the vectors in S are linearly independent if

   $$x_1\alpha_1 + \ldots + x_N\alpha_N = \vec{0}$$

   implies

   $$x_1 = x_1 = \cdots = x_N = 0.$$
9. The definition of an inner product on a vector space:

   If V is a vector space over the scalars S, a subset of the complex numbers, then $(|) : V \times V \to S$ is called an inner product on V if

   $$\begin{aligned} (\alpha + x\beta|\gamma) &= (\alpha|\gamma) + x(\beta|\gamma) \\ (\nu|\nu) &> 0 \\ (\alpha|\beta) &= \overline{(\beta|\alpha)} \end{aligned}$$

   for all vectors $\nu \neq \vec{0}$, α, β and γ and all scalars x.
10. The definition of orthogonal:

    The vectors α and β are orthogonal if and only if $(\alpha|\beta) = 0$.
11. The following facts about symmetric matrices:
    (a) A square matrix A is called symmetric if $A_{i,j} = A_{j,i}$ for all pairs of indices (i, j).
    (b) The eigenvalues of a real symmetric matrix are real numbers.
    (c) The eigenvector of a real symmetric matrix are orthogonal if they belong to different eigenvalues.