

MthStat 465, Spring 2005, Midterm Exam

You may use paper, pencil/pen, ruler and simple calculator.

- (5 points) A complete set of events in a sample space is called a *sigma algebra*. What three properties must a set of events have in order to be a sigma algebra?
- (10 points) Give a full model (Sample space, sigma algebra, probability measure) to describe the following: A fair coin is tossed twice. Explicitly list the samples space, all the events, and the rule by which the probability of an event is assigned. Which event describes “the number of heads equals the number of tails”?
- (10 points) Use the identity $(A \cap B)^c = A^c \cup B^c$ to conclude that the intersection of two events is also an event.
- (15 points) What is meant by geometric probability? Give three examples, one with length, one with area, and one with volume.
- (10 points) Derive the formula

$${}_n C_k = \frac{n!}{k!(n-k)!}$$

for integers n and k satisfying $0 < k \leq n$.

- (10 points) Why is it better to define the independence of A and B by requiring $\Pr(A \cap B) = \Pr(A)\Pr(B)$ than $\Pr(A|B) = \Pr(A)$? Suppose that $\Pr(A) > 0$, $\Pr(B) >$ and A and B are independent. Show that $A \cap B \neq \emptyset$.
- (10 points) Describe the relative merits of using the mean and median to describe the center of a data set.
- (10 points) Use the data $(1, 2)$, $(3, 5)$, $(4, 7)$, $(5, 2)$, $(6, 7)$, $(8, 4)$, $(9, 2)$, $(9, 3)$, $(10, 5)$ to illustrate an explanation of the construction of the median-median line of a data set.
- (10 points) What is the least squares criterion for fitting a curve to a bivariate data set? According to this criterion, which of the following lines is a better fit to $(1, 3)$, $(2, 5)$, $(3, 1)$: $y = x + 1$ or $y = 2x - 1$. Justify your answer.
- (10 points) Give three ways to measure the spread of a data set. Illustrate them with the data 1, 2, 3, 4, 5.