

MthStat 465, Spring 2005, Homework Number 4

Some routine computations. In each case show all the work and give all the explanations needed to justify your answers.

1. A box contains 15 light switches. 5 are defective and 10 work correctly. If 6 switches are selected from the box, what is the probability that x of the switches is defective, for $x = 0, 1, 2, 3, 4, 5$?
2. A coin with probability $1/3$ of coming up heads is tossed 25 times. What is the probability that it comes up heads more than 15 times? Give a decimal approximation.
3. Suppose that N has the standard normal distribution, and $R = 5N - 7$. Use a table to find $\Pr(-9 < R < -6)$.
4. Suppose that the mean of X is 10 and the variance of X is 2. What are the mean and variance of $-3X + 5$?
5. A fair coin is tossed until the first head appears. What is the probability that x or more tosses are required, for x any positive integer?
6. $\Pr(A) = 1/4$, $\Pr(B) = 1/7$ and $\Pr(A \cup B) = 5/14$. Are A and B independent?
7. The ordered pair (R, S) is uniformly distributed on the square $\{(r, s) : 0 \leq r \leq 1, 0 \leq s \leq 1\}$. Find the distribution function of $R - S$.
8. The random variable Θ is uniformly distributed on $\{k\pi/6, k = 0, 1, \dots, 11\}$, and $R = \cos(\Theta)$ and $S = \sin(\Theta)$. Find the probability mass functions of R and S , and compute $E[R]$, $E[S]$ and $E[RS]$. Are R and S independent?
9. Suppose that a fair coin is tossed 1000 times. Use the DeMoivre-Laplace theorem to estimate the probability that the number of heads is between 475 and 525.