MthStat 465, Spring 2005, Homework Number 3

1. Graph the function
$$f(x) = \begin{cases} 0 & \text{if} \quad x \le -1 \\ 1 - |x| & \text{if} \quad -1 \le x \le 1 \\ 0 & \text{if} \quad 1 \le x \end{cases}$$
and verify that it is a density function. Find the corresponding distribution function. If R is a random variable with this distribution function, compute $\Pr(-1/3 < R \le 2/3)$. Use the graph of the density function to support your answer.
2. Show that $f(x) = (2\pi)^{-1/2} e^{-x^2/2}$ is a density function. Draw its graph, labeling clearly the points of inflection. Suppose that N is a random variable with this density. Without computing anything, explain why $\Pr(N \le t) = \Pr(N \ge -t)$ for any real number t, and that $\Pr(N \le 0) = 1/2$. Use the trapezoid rule to show that $\Pr(|N| \le 1) \approx 0.68$, $\Pr(|N| \le 2) \approx 0.95$, and $\Pr(|N| \le 3) \approx 1.00$.
3. Suppose that R has a uniform distribution on $(0, 1)$. Show that the distribution function of $S = -\ln(1 - R)$ is
$$F_S(t) = \begin{cases} 0 & \text{if} \quad t \le 0 \\ 1 - e^{-t} & \text{if} \quad t > 0 \end{cases}$$
Show that for positive real numbers a and b,
$$\Pr(S > a + b | S > a) = \Pr(S > b).$$
Find the mean and variance of S.
4. Suppose that R and S are two random variables, and
$$\begin{aligned} \Pr(R = 1, S = 1) &= 1/15 \\ \Pr(R = 1, S = 2) &= 2/15 \\ \Pr(R = 2, S = 1) &= 3/15 \\ \Pr(R = 2, S = 2) &= 4/15 \\ \Pr(R = 3, S = 4) &= 5/15 \end{aligned}$$
Find the probability mass function of S and of R. Graph the probability distribution function of R and of S. Determine if R and S are independent random variables, and justify your answer. Find $\Pr(R = 1|S = 2)$.
5. Show that if A and B are independent, then so are A and B^c. Show that if $\Pr(A) = 0$ then A is independent of any other event. Show that if $\Pr(A) > 0$ and $\Pr(B > 0)$ and A and B are independent, then $A \cap B \ne \emptyset$.
6. Derive the general formula for the mean and for the variance of binomial random variables.