

The Fundamental Theorem of Calculus

The capstone result of our study of differentiation and integration is the Fundamental Theorem of Calculus. We will not state the most general version but give a version that is sufficient for most purposes. It provides a link between differentiation and integration.

Theorem 1 *Suppose that F is continuous on $[a, b]$ and $F'(x) = f(x)$ on (a, b) . If f has an integral on $[a, b]$ then we have*

$$\int_a^b f(x) dx = F(b) - F(a).$$

Furthermore, if f is continuous on $[a, b]$ then for each $x \in (a, b)$,

$$\frac{d}{dx} \int_a^b f(u) du = f(x)$$

For example, the Fundamental Theorem of Calculus tells us that every continuous function on $[a, b]$ has an antiderivative on (a, b) . We have given a proof of this theorem under the additional condition that f is monotone. The proof in the general case is very similar.