Exam 01 Study Guide

This exam will focus on derivatives and their applications. You should be prepared to state definitions and theorems, and apply them in straightforward situations that you have already seen in class.

Exam 1 Material: You are responsible for all material on Exam 1. See

`http://www.uwm.edu/~ericskey/221material/exam01topics.pdf`

for details.

In particular, you should be able answer any problem from Exam 1 that you did not answer correctly on that exam.

Mean Value Theorems: You should be able to state Rolle's Theorem and use it to derive the Mean Value Theorem:

> If $f : [a, b] \to (-\infty, \infty)$ is continuous on $[a, b]$ and differentiable on (a, b) then there is some $z \in (a, b)$ so that $f(b) - f(a) = f'(z)(b - a)$.

and the Generalized Mean Value Theorem:

> If $f : [a, b] \to (-\infty, \infty)$ and $g : [a, b] \to (-\infty, \infty)$ are continuous on $[a, b]$ and differentiable on (a, b) then there is some $z \in (a, b)$ so that
>
> $$g'(z)(f(b) - f(a)) = f'(z)(g(b) - g(a)).$$

You should be able to use the Mean Value Theorem to explain how the sign of the first derivative of a function relates to the increasing/decreasing behavior of the function.

Error Formulas: You should know the error formulae for the best linear approximation:

$$f(x) - (f(a) + f'(a)(x - a)) = \frac{1}{2} f''(z)(x - a)^2$$

for some z between a and x;

and for the linear interpolation formula: If $a < x < b$ then

$$f(x) - \left(f(a)\frac{b - x}{b - a} + f(b)\frac{x - a}{b - a} \right) = -\frac{1}{2} f''(z)(b - x)(x - a)$$

for some $z \in (a, b)$.

You should be able to use these theorems to explain the relation of the sign of the second derivative of a function to the shape of its graph. In particular, you should be able to define what it means for the graph of a function to be convex.

L'Hopital's Rule: You should be able to state and correctly apply L'Hopital's Rule for limits of quotients.

Newton's Method: You should be able to explain Newton's Method of Approximation for solving $f(x) = 0$ and give the conditions under which it is applicable in the context of a specific example.

Optimization Problems: You should be able to use graphical methods, supported by calculus, so solve optimization problems.

Graphing: You should be able to construct graphs indicating both monotinicity and convexity/concavity of the graph.

Related Rates: You should be able to solve simple related rates problems.

Inverse Functions: You should be able to state the conditions under which inverse functions are differentiable and to use the chain rule to find their inverses.

Differential Equations: You should be able to solve problems of the form

$$\begin{aligned} f''(x) + k^2 f(x) &= 0 \\ f(0) &= A \\ f'(0) &= B \end{aligned}$$

and

$$\begin{aligned} f'(x) + k f(x) &= g(x) \\ f(0) &= A. \end{aligned}$$

You are **NOT** responsible for the physics giving rise to such equations, but you should be able graph the solutions of these equations.