Exam 01 Study Guide

This exam will focus on limits, continuity and derivatives. You should be prepared to state definitions and theorems, and apply them in straightforward situations that you have already seen in class.

1 Prerequisite Material

You are responsible for all course pre-requisite material. This includes the information found at the following links:

`http://www.mhhe.com/math/precalc/barnettcat7/student_index.mhtml`

`http://www.uwm.edu/~ericskey/231material/alg.pdf`

`http://www.uwm.edu/~ericskey/231material/trig.pdf`

`http://www.uwm.edu/~ericskey/231material/functions.pdf`

2 Notes on line:

You are responsible for the material found at

`http://www.uwm.edu/~ericskey/221material/module01.pdf`
`http://www.uwm.edu/~ericskey/221material/module02.pdf`

and the material in

`http://www.uwm.edu/~ericskey/221material/module03.pdf`

that has been covered as of September 23, 2008, with the following general exception: you will not be expected to reproduce the reasoning, just the results.

3 Specifics:

Give special attention to the following.

1. Whenever a definition is called for, it will be the version from the class notes. Informal definitions will get zero credit. For example, an **infinite sequence** is a function whose domain is the non-negative or positive integers.

2. You will be expected to state the definition of limit at infinity and to explain the relation of this definition to the concept of horizontal asymptote.

3. You will be expected to give a formal proof that

$$\lim_{n\to\infty} \frac{1}{x} = 0$$

   and a formal proof that

$$\lim_{n\to\infty} \frac{1}{2^n} = 0.$$

   See the lecture notes for these proofs.

4. You are expected to know the definition of the natural exponential function exp:

$$\lim_{n\to\infty} \left(1 + \frac{r}{n}\right)^n = \exp(r),$$

   its properties, and its relation to e and e^r.

5. You are expected to know the definition of the natural logarithm function and its properties.

6. You are expected to know the following specific limits derived from Pinching:

$$\begin{aligned}\lim_{x\to 0}\frac{\exp(x)-1}{x} &= 1\\ \lim_{x\to 0}\frac{\ln(1+x)}{x} &= 1\\ \lim_{x\to 0}\frac{\sin(x)}{x} &= 1\\ \lim_{x\to 0}\frac{1-\cos(x)}{x^2} &= \frac{1}{2}\end{aligned}$$

7. You should be able to apply the Pinching Principle if you are given the bounding functions $f(x)$ and $h(x)$ for which we know $f(x) \le g(x) \le h(x)$. You should be able to explain the Pinching Principle graphically.

8. You are expected to be able to state the definition of continuous function and demonstrate that piecewise defined functions are continuous with and without the use of a graph.

9. You will be expected to state the definition of differentiable:

   A function f is differentiable at a if there is some constant m so that

$$\lim_{x\to a}\left|\frac{f(x)-(f(a)+m(x-a))}{x-a}\right| = 0$$

   and the number m is called the derivative of f at $x = a$,

   and to give the three interpretations of derivatives, along with examples.

10. You will be expected to be able to precisely state the Chain Rule, Product Rule, Quotient Rule, and the rule for derivatives of sums and constant multiples, and to apply them in simple circumstances. You should be able to prove the rule for sums and constant multiples.

11. You are expected to know the relation between differentiability and continuity, and to prove it.

12. You should be able to use the definition of derivative to determine where a piecewise defined function is differentiable.

13. You are expected to know (but not derive) the following limits:

$$\begin{aligned}\lim_{h\to 0}\frac{\exp(h)-1}{h} &= 1\\ \lim_{h\to 0}\frac{\ln(1+h)}{h} &= 1\\ \lim_{h\to 0}\frac{\sin(h)}{h} &= 1\\ \lim_{h\to 0}\frac{1-\cos(h)}{h^2} &= \frac{1}{2}\end{aligned}$$

14. You will be expected to be able to derive the following:

$$\begin{aligned}\exp'(x) &= \exp(x)\\ \ln'(x) &= 1/x \text{ if } x>0\\ \sin'(x) &= \cos(x)\\ \cos'(x) &= -\sin(x)\end{aligned}$$

    using the definition of derivative and the limits in the preceding item.

15. You are expected to be able to use the quotient and product rules to derive the formulas for the derivatives of tan(x), cot(x), sec(x) and csc(x).

16. You should be able to solve problems involving tangent lines, such as:

(a) Find an equation of a line tangent to the graph of y = f(x) through a point NOT on the graph of y = f(x).

(b) Find the location of lines tangent to y = f(x) with specified slopes, such as: Find all lines tangent to $y = 2x^3 + 3x^2 + 7$ whose slopes are 1.

17. You should be able to solve rate problems.

18. You should be able to give approximations to functions using best linear approximations.

19. You should be able to explain how Newton's method of approximation works in terms of zeros of linear approximations, and use it to find an approximate solution to a simple polynomial equation.