Abstracts

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A variation on finite type classical knot invariants

James Conant

In joint work with Ted Stanford and Jacob Mostovoy, we analyze finite type knot invariants where the notion of "crossing change" is replaced by "simple null move." A simple null move is equivalent to tying a copy of the Borromean rings into three pairs of antiparallel strands of a knot. Results of Garoufalidis Kricker and Rozansky imply that the only type one and type zero rational-valued invariants come from the Alexander polynomial. We will show that more interesting behavior occurs for torsion invariants.

Homotopy equivalence in 1-dimensional and planar Peano continua

Greg Conner

We will discuss joint work with Grad students Mark Meilstrup and Brent Gorbutt concerning how to find the "simplest" representative of a given homotopy type of 1-dimensional Peano continua.

In joint work with Cannon and Zastrow we gave an example of a planar Peano continuum which is not homotopy equivalent to a 1-dimensional space. In this talk we will discuss joint work with Cannon which provides a topological description of which planar Peano continua have this property.

Extension vs. Selection Properties

Tadek Dobrowolski

The relationship between the extension property and the selection properties for convex sets will be discussed.

Conjectures and speculations concerning actions by Cantor groups on ENRs

Robert D. Edwards

A *cantor group* is a topological group whose underlying space is homeomorphic to the cantor set, i.e. is totally-disconnected perfect compact metrizable. Equivalently, a cantor group is an infinite second-countable profinite group, where *profinite* is short for *projectively finite*, meaning a projective (i.e. inverse) limit of finite groups. Examples of such groups are (a) a direct product of countably infinitely many (nontrivial) finite groups, and (b) the inverse limit of the groups $\mathbf{Z}/p^k\mathbf{Z}$, for a fixed prime p and increasing k, under the natural (essentially unique) bonding epimorphisms. Type (a) provides examples (not unique!) of universal cantor groups, while type (b) is the cantor group known as the p-adic integer group, or p-adic integers, for short.

This talk concerns my efforts (long-term!) to understand how cantor groups can act on reasonably nice spaces. The basic overarching conjecture in this area is the

Free-Set Z-set Conjecture: The free set of a cantor group action on an ENR is a homology Z-subset of the ENR.

I will discuss the status of this conjecture, and offer some thoughts on its hitherto undiscussed "small torsion" aspects, which involve some interesting adaptations of some classical constructions.

Recognizing open subsets of euclidean space

Steve Ferry

We describe an obstruction theory which decides which open manifolds are complements of tame k-dimensional UV^1 compacta in S^n, $2k+2<n$.

Generalized universal covering spaces and the shape group

Hanspeter Fischer (joint with Andreas Zastrow)

We present a condition under which a path connected topological space X admits a generalized universal covering space, even if it is not semilocally simply connected. The condition is that the natural homomorphism $\varphi : \pi_1(X) \to \check{\pi}_1(X)$ from its fundamental group into its first shape homotopy group be injective.

This generalized notion of universal covering $p : \tilde{X} \to X$ at which we arrive, is universally characterized by the following three properties:

(1) $\tilde{X}$ is path connected, locally path connected and simply connected;
(2) $p : \tilde{X} \to X$ is a continuous surjection;

(3) for every continuous $f : (Y, y) \to (X, x)$, with Y path connected, locally path connected and simply connected, and for every $\tilde{x}$ in $\tilde{X}$ with $p(\tilde{x}) = x$, there exists a *unique* continuous lift $g : (Y, y) \to (\tilde{X}, \tilde{x})$ with $f = p \circ g$.

Additional properties of this generalized universal covering include:

(i) $\mathrm{Aut}(\tilde{X} \xrightarrow{p} X) \equiv \pi_1(X)$;
(ii) $p : \tilde{X} \to X$ is open if and only if X is locally path connected;
(iii) if X is locally path connected and semilocally simply connected, then $p : \tilde{X} \to X$ agrees with the usual universal covering.

For many spaces, semilocally simply connected or not, $\varphi : \pi_1(X) \to \check{\pi}_1(X)$ is known to be injective: for example, for all subsets of the Euclidean plane, for all 1-dimensional compacta, as well as for boundaries of certain Coxeter groups. Consequently, our result provides these classes of spaces with a generalized universal covering, which enjoys most of the usual properties with the possible exception of evenly covered neighborhoods.

Intrinsically linked graphs with knotted components

Tom Fleming

We construct a graph G such that any embedding of G into the three sphere contains two disjoint cycles that form a nonsplit link. Further, at least one of these two cycles is shown to be a nontrivial knot.

Rigid Cantor sets with simply connected complement

Dennis Garity (joint with D. Repovs and M. Zeljko)

It will be presented that there exist uncountably many inequivalently embedded rigid wild Cantor sets in $\mathbb{R}^3$ with simply connected complement. Previous constructions of wild Cantor sets in $\mathbb{R}^3$ with simply connected complement, in particular the Bing-Whitehead Cantor sets, had strong homogeneity properties. This suggested it might not be possible to construct such sets that were rigid. The examples given are constructed using a modification of a construction of Skora together with a careful analysis of the local genus of points in the Cantor sets.

Open manifolds homotopy equivalent to finite complexes

Craig Guilbault

Last winter I was asked the following question by Igor Belegradek:

Question. *Let M be an open manifold homotopy equivalent to an embedded compact submanifold, say a torus. Is $M \times \mathbb{R}$ homemorphic to the interior of a compact manifold?*

I was surprised to realize that I could not give an immediate answer—and that an answer did not seem to exist in the literature. I will discuss the eventual solution to a generalized version of this question in dimensions ≥ 5.

Positive Dehn Twist Expressions for Some Finite Order Elements in the Mapping Class Group

Yusuf Gurtas

The talk will present explicit positive Dehn twist expressions for some elements of finite order in the mapping class group of 2-dimensional closed, compact, oriented surfaces. The computer results for the computation of the signatures of the symplectic Lefschetz fibrations that they define will also be presented and a closed formula for their signature will be conjectured.

Approximating cell-like maps by cellular maps in dimension 5

Denise Halverson (joint with Bob Daverman)

Suppose that G is an upper semicontinuous cell-like decomposition of a 5-manifold M such that M/G is finite dimensional and has the disjoint disks property. We show that the associated decomposition map $\pi : M \to M/G$ can be approximated by a cellular map $f : M \to M/G$. In particular, the nondegeracy set N_f will have embedding dimension ≤ 2.

The 'Good' set in a 1-dimensional Peano continuum is a locally finite graph

Mark Meilstrup

For 1-dimensional Peano continua, we define the 'Good' set to be those points with neighborhoods containing no non-contractible closed curves. We prove that this 'Good' set is actually a locally finite graph for reduced 1-dimensional Peano continua.

On Singer Conjecture for Coxeter groups

Boris Okun

A conjecture of Singer states that the reduced L^2-cohomology of the universal cover of an aspherical manifold vanishes except possibly in the middle dimension. Associated to any finite flag complex L there is a right-angled Coxeter group W_L and a contractible cubical complex Σ_L (the Davis complex) on which W_L acts properly and cocompactly, and such that the link of each vertex is L. It follows that if L is an $(n-1)$-sphere, then Σ_L is a contractible homology n-manifold. We prove the Singer Conjecture (and its generalization for the weighted L^2_q-cohomology) for Σ_L when L is the barycentric subdivision of a triangulation of a $(2k-1)$-sphere.

Can the four sphere decompose upper semicontinuously into circle-like compacta?

David Snyder

The structure of locally smooth actions of the circle on 4-manifolds was characterized by R. Fintushel in the mid to late 1970's. On the 4-sphere, such an action always has at least one singular orbit. A question naturally arises: is there any continuous map defined on the 4-sphere such that all point inverses are circle-like? We address this question and related issues.

New obstructions for embedding 2-spheres into 4-manifolds

Peter Teichner

In joint work with Rob Schneiderman, we have developed a new obstruction theory for the embedding problem for 2-spheres in 4-manifolds. It is given in terms of the intersection theory of Whitney towers, immersed in the 4-manifold, and it is related to Milnor invariants and the Kontsevich integral in the easiest cases (where the 4-manifold is given by attaching 2-handles to a link in the 3-sphere). As a consequence, we give an intersection theoretic explanation of the Milnor invariants, and we relate them to the existence of embedded gropes in the 4-ball. In this sequence of talks, we shall give an outline of the theory, explain the main results, and discuss the remaining open problems. There are 3 papers, all joint with Rob Schneiderman (and one also joint with Jim Conant) available on my homepage: `https://www.math.ucsd.edu/~teichner/`

2-complexes with self covers

Mat Timm

Finite connected regular 2-complexes that nontrivially cover themselves can be completely classified. We will do so.

Manifolds with non-stable fundamental group at infinity, III

F.C.Tinsley (joint with C.R.Guilbault)

We succeed in generalizing Siebenmann's characterization of collarable ends to the setting in which the fundamental group at infinity fails to be stable. We establish necessary and sufficient conditions for an inward tame end to have arbitrarily small homotopy collar neighborhoods of infinity (i.e., be pseudo-collabable). First, the fundamental group at infinity must be perfectly semistable. Second, the generalized Wall obstruction must vanish. We also investigate the situation in which the inward tameness condition is weakened slightly.

Normal Curves in 2-manifolds, a preliminary report

Bobby Winters

This is a report on joint work with Robert Myers concerning least weight curves in triangulated 2-manifolds. The aim is to copy the results obtained by Freedman, Hass, and Scott in their paper on shortest curves and to extend them to the noncompact case. This is done in hopes of providing a paradigm to follow for noncompact normal surfaces in 3-manifolds.