Stat 761 **Fall 2005**

Poisson Processes

$o(\cdot)$ **notation:** We say that a function $f(x) = o(x)$ as $x \to a$ if $\lim_{x\to a} f(x)/x = 0$. Note that $o(x) + o(x) = o(x)$ and $c \cdot o(x) = o(x)$ for any constant c.

Counting processes: A *counting process* is a process in which distinct events occur over time (or space or ...), where the number of events in a fixed time period is a random variable. (Also called a *point* or *jump* process.)

Examples:

- radioactive particle emissions
- customers arriving at a queue
- earthquakes at a given location
- a certain action or behavior of fish in a tank
- defects in a wire (t = length)
- defects in cloth (t = area)
- raisins in cake (t = volume)

Axioms of a Poisson process: A counting process is a *Poisson process* if there is a constant $\lambda > 0$ such that, as $h \to 0$,

1. P(1 event in an interval of length h) $= \lambda h + o(h)$.
2. P(at least 2 events in an interval of length h) $= o(h)$.
3. The only other possibility is 0 events.
4. If I and J are disjoint intervals, then the number of events occurring in I and the number occurring in J are independent.

The constant λ is known as the *intensity* of the process.

Theorem P *For each t, let N_t = the number of events in the interval $(0, t)$ in a Poisson process. Then $N_t \sim$ Poisson(λt). In particular, $N_1 \sim$ Poisson(λ).*

Thus λ is the *mean number of events per unit time.*