Stat 761 **Fall 2005**

Assignments for Chapter 5

Section	Problems
5.1, 5.2	6b, 11*
5.3	18(b)*, 19*, 8.41(b)
5.4	21, 24
5.5.1	39(a)*, A21
5.5.3	41*, A22, A23
5.5.2	A24, A25
5.5 ($\xrightarrow{r}$)	A26
5.5.4	44
5.6	56*, 62*

Additional Problems:

A21: Prove: $U_n \xrightarrow{P} c$, $V_n \xrightarrow{P} d \Rightarrow U_n + V_n \xrightarrow{P} c + d$. Use the ordinary triangle inequality; also, $A + B \geq \epsilon \Rightarrow A \geq \epsilon/2$ or $B \geq \epsilon/2$.

A22: Let $\hat{p}_n$ be the sample proportion of successes in the first n of a sequence of independent Bernoulli(p) trials, and $\hat{q} = 1 - \hat{p}$. Show that $(\hat{p}_n - p)/\sqrt{\hat{p}_n\hat{q}_n/n} \xrightarrow{D} N(0,1)$.

A23: Let $Z_n \sim$ Poisson(n), and let $Y_n = (Z_n - n)/\sqrt{n}$. Show that $Y_n \xrightarrow{D} N(0,1)$.

A24: Let $\{X_n\}$ be a sequence of distinct r.v's having the exponential(1) distribution. Show that $\mathrm{P}(X_n > n \text{ i.o.}) = 0$. Show that $X_n/n \to 0$ a.s.

A25: Prove: If $X_1, X_2, \ldots$ are i.i.d. with finite 4th moment and mean μ, then $\overline{X}_n \to \mu$ a.s. *Hint:* Use the Markov inequality with the exponent 4. Expand $[n^{-1}\sum_{i=1}^{n}(X_i - \mu)]^4$ before computing its expected value. You actually only need certain terms in the expanded sum, since a lot of terms will have expectation zero. Apply Borel-Cantelli.

A26: Prove: If $X_n \xrightarrow{r} X$ for some $r \geq 1$ then $X_n \xrightarrow{P} X$.

*Notes, hints:

5.11: Assume that if $\sigma > 0$ then S^2 has a non-degenerate distribution. (See ECR7.)

5.18(b): Easiest using the "method of random variables".

5.19: (a) and (b) Easiest using the "method of random variables" rather than working with the cdf's.

(b) $F_{k,\nu}$ denotes a random variable with this distribution. *Note:* The book's method is wrong – (a) doesn't apply.

(c) $F_{\alpha,k,\nu}$ denotes the $(1-\alpha)$th quantile of this distribution.

5.39a: The hint assumes that h is uniformly continuous. You may make this assumption. (See ECR8.)

5.41: An alternative to the book's hint is to rewrite what you can in terms of the cdf's $F_n(x)$. (For full credit, make no assumptions about the distribution of X_n or the existence of moments. Note minor typo: In the second limit, we should have $x > \mu$, not $\geq$. Theorem 5.5.13 states this correctly.)

5.56: The pdf is missing the factor $1/\pi$, and the cdf is incorrect.

5.62: In (a) and (b), use the standard distributions (Cauchy(0,1) and double exponential(0,1) – see p. 623.)

Extra Credit Problems:

ECR7 Let S^2 be the sample variance from a sample $X_1, \ldots, X_n$ with $\mathrm{Var}(X_i) = \sigma^2$. Prove: If $\sigma > 0$, then S^2 does not have a degenerate distribution.

ECR8 Problem 5.39a, assuming only continuity.

ECR9 Suppose $X_n \geq 0$ a.s., and $X_n \xrightarrow{P} a$. Show that $a \geq 0$.

ECR10 Define $X_0 = 0$, $X_1 = 1/2$, and

$$X_{n+1} = \begin{cases} X_n/2 & \text{with probability } 1/2, \\ (X_n+1)/2 & \text{with probability } 1/2. \end{cases}$$

Prove that $X_n \xrightarrow{D}$ the uniform distribution on $(0,1)$. (Prove that $\mathrm{P}(a < X_n < b) \to (b-a)$.)