Stat 761 **Fall 2005**

Assignments for Chapter 3

Section	Problems
3.2	2a, 6*, 11a*, A11, A12
3.3	17, A13
3.4	28(b*,d), 29d*, 30a, 31*
3.5	42*
3.6	44, 45(a,b)*, 46

***Notes:**

Problem 6: *Change the problem* as follows: Suppose that there are 200 insects. In (b) and (c),"fewer than 4". Use the *exact* model in (a) and (b).

Problem 11: Prove rigorously (don't just argue by using "$\approx$"). Assume $0 < p < 1$.

Problem 28b: Just do the "both unknown" case.

Problem 29d: Also find the function $c^*(\eta)$.

Problem 31: Note the typos (missing dx, $\int$). Part (a) should read: Start from the equality

$$\int f(x|\boldsymbol{\theta})dx = 1,$$

replace f by its exponential form (3.4.1), differentiate both sides, and then rearrange terms to establish (3.4.4). *You may assume* that the functions $c(\boldsymbol{\theta})$ and $w_i(\boldsymbol{\theta})$ are differentiable and that it is permissible to interchange integration and differentiation.

Problem 42: "Sample space" here means "range" or "support". In Stat 762 we will see that "stochastically increasing" is related to the concept of "monotone likelihood ratio" arising in the theory of hypothesis testing (Sec. 8.3).

Problem 45: You can apply the Markov inequality.

Additional Problems:

A11: Complete the proof of *Theorem P* (see handout on "Poisson Processes"). Specifically, prove that $\mathrm{P}(N_t = 1) = e^{-\lambda t}\lambda t$ and (by induction) that $\mathrm{P}(N_t = k) = e^{-\lambda t}(\lambda t)^k/k!$.

A12: An urn contains N objects, numbered 1 through N. A sample of n objects are chosen without replacement. Associate with each object a Bernoulli random variable so that

$$X_i = 1 \text{ if object } i \text{ is in the sample,}$$

and $= 0$ otherwise, $i = 1, \ldots, N$.

Assume that M objects in the urn are of type "A", the rest, "B". Associate to the i-th object a (nonrandom) number a_i such that

$$a_i = 1 \text{ if object } i \text{ is of type A,}$$

and $= 0$ otherwise, $i = 1, \ldots, N$.

a. Find $\sum_{i=1}^{N} a_i$ and $\sum_{i=1}^{N} X_i$.

b. Find $\mathrm{E}(X_i)$.

c. Find $\mathrm{E}(\sum_{i=1}^{N} a_i X_i)$.

d. Let $Y = \sum_{i=1}^{N} a_i X_i$. Explain why Y has a hypergeometric distribution. (What does it count?)

A13: Let T = time until the k-th event in a Poisson process with intensity λ. Show that T has a gamma distribution; what are α and β? (Hint: You may quote the result given in problem 3.19.)

ECR2. Let X have natural exponential density (3.4.7). Show that the function c^* is differentiable. You may use Theorem 2.4.3.