Stat 761 **Fall 2005**

Assignments for Chapter 2

Section	Problems
2.1	3, 6a, 7a, 8b, A6, 17, A7
2.2	11b, 14, 18, 26(b,c), A8
2.3	30c*, 32, A9, A10
2.4	Just read.

Additional Problems:

A6: Show that the infimum in equation (2.1.13) is really a minimum, that is, that it is actually attained by some x in the set.

A7: Find a formula for the p-th quantile x_p for the distributions in Problem 2.17.

A8: Theorem *Let X be a random variable with pdf/pmf $f(x)$, let $g(x)$ be a (measurable) function, and let $Y = g(X)$. Then*

$$\mathrm{E}(Y) = \begin{cases} \sum_x g(x) f_X(x) \\ \int_{-\infty}^{\infty} g(x) f_X(x) dx \end{cases} \tag{1}$$

Prove this theorem

a. where X is discrete and g is one-to-one.

b. where X is continuous and g is differentiable and strictly monotonic.

A9: Let X have the gamma distribution with parameters α and β. Find the m.g.f. of $aX + b$. For what value(s) of a and b (if any) does it also have a gamma distribution?

A10: Prove: $|\mathrm{E}(Y)| \leq \mathrm{E}|Y|$, with equality iff either $Y \geq 0$ a.s. or $Y \leq 0$ a.s. (**Hint:** Let $Y^+ = \max(Y, 0)$ and $Y^- = \min(Y, 0)$. Express both Y and $|Y|$ in terms of Y^+ and Y^- and apply the properties of E. You don't need to prove that Y^+ and Y^- are random variables (they are!).)

***Notes:**

Problem 2.30c: Include the domain of $M(t)$. Your derivation should explain the domain.

Extra Credit problems: Optional. Do each on a separate sheet. You may hand in an extra-credit problem at any time during the semester, until the last class. You may do a problem over for increased credit (up to 10 points, applied towards your homework grade).

ECR1 Let F be a cdf. Fix $y \in (0, 1)$, and let $B = \{x : F(x) \geq y\}$. Show that B and B^c are nonempty. Show that every element of B^c is a lower bound of B.