Stat 761 **Fall 2005**

Assignments for Chapter 1

Notes: 1. You are responsible for reading the text. Problems may occasionally cover material not presented in lecture.

2. Show enough work to indicate your thinking. This may mean giving the relevant computation, or describing your logical reasoning. Answers alone are generally insufficient.

3. The text does not have an index of notation. A good idea is to create your own, particularly every time you see notation which you feel is non-standard and easy to forget.

Section	**Problems**	
1.1	11c, A1	
1.2.1-1.2.2	4, 12a*, A2	
1.2.3-1.2.4	20*	
1.3	23, 24*, 33, A3, A4	
1.4	45, A5	
1.5	47 (except (b)), 48*	(47(b) is done in the text)
1.6	51, 54	

Additional Problems:

A1: Is the union of two sigma fields on S a sigma field on S? (Prove or give a counterexample.)

A2: Suppose $A_1 \supset A_2 \supset \cdots$ and $A = \cap A_i$. Show that $\mathrm{P}(A_n) \to \mathrm{P}(A)$. (This is one of the "continuity properties" of probability. When $A = \emptyset$, this is sometimes assumed as an axiom – see Page 39.)

A3: At a carnival, a man gives John a chance to win \$1 as follows. He shows John three cards, one white on both sides, one red on both sides, and the third white on one side and red on the other. He places the cards in a container and asks John to shuffle them and (without looking) to withdraw one and place it on his table. John does this, and places a card on the table whose top face is red. The man then bets \$1 to John's \$1 that the bottom face is also red. Should John accept the bet? (Hint: See the prisoner's dilemma and car-and-goats problems.)

A4: Give your own example of sets A, B, and C for which

$$\mathrm{P}(A|B) < \mathrm{P}(A|B^c)$$

but

$$\mathrm{P}(A|B \cap C) > \mathrm{P}(A|B^c \cap C) \quad \text{and} \quad \mathrm{P}(A|B \cap C^c) > \mathrm{P}(A|B^c \cap C^c).$$

(This is a formulation of *Simpson's paradox*.)

A5: Prove parts (b) and (c) of the inverse image theorem (next page). (Part (a) is similar.)

***Notes, hints:**

Problem 1.12a: Hint: Write a finite union of disjoint sets as a countable union.

Problem 1.20: The answer without order is .0249, to 4 decimals (text assumes order matters).

Problem 1.24: The book's hint in (c) seems to apply to all parts of the problem.

Problem 1.48: You will need the inverse image theorem (next page) and the "continuity" property of probabilities.

About inverse images

Let $f : A \to B$ be a function, and let $f^{-1}(C) = \{x : f(x) \in C\}$, the *inverse image* of C.

Theorem 1 *If C_i, C and D are subsets of B, then*

a. $f^{-1}(\cup_i C_i) = \cup_i f^{-1}(C_i)$

b. $f^{-1}(\cap_i C_i) = \cap_i f^{-1}(C_i)$

c. $C \subseteq D$ *implies* $f^{-1}(C) \subseteq f^{-1}(D)$

The same doesn't hold for *images* $f(E)$; in particular, part (b) is only an inclusion, in general. Thus inverse images "behave better" than images.