Math Stat 465

Some notes on expectation with joint distributions

Suppose that X and Y are **discrete** random variables with joint pmf $p(x, y)$. Then for any function h of X and Y,

$$\mathrm{E}(h(X,Y)) = \sum_x \sum_y h(x,y)p(x,y). \tag{1}$$

In particular, $\mathrm{E}(X) = \sum_x \sum_y xp(x,y)$ and similarly for $\mathrm{E}(Y)$. However, it is often easier to calculate the marginal pmfs p_X and p_Y first and then to use the original rules for expected value:

E1. $\mathrm{E}(X) = \sum_x xp_X(x)$

E2. $\mathrm{E}(Y) = \sum_y yp_Y(y)$.

For **continuous** random variables, equation (1) and rules E1 and E2 hold with the following changes: pmfs are changed to pdfs, and sums are changed to integrals.

For **both the discrete and continuous cases**, the following rules apply:

E3. $\mathrm{E}(g(X,Y) + h(X,Y)) = \mathrm{E}(g(X,Y)) + \mathrm{E}(h(X,Y))$.

E4. $\mathrm{E}(cg(X,Y)) = c\mathrm{E}(g(X,Y))$.

E5. If X and Y are independent, then $\mathrm{E}(XY) = \mathrm{E}(X)\mathrm{E}(Y)$.