Math Stat 465

Covariance and Correlation: Example

We have considered the following example: X and Y have joint pmf $p(x, y)$ given by the following table:

		y = 0	1	$p_X(x)$
x	0	.1	.2	.3
	1	.2	.1	.3
	2	0	.4	.4
	$p_Y(y)$	.3	.7	

From this table we computed the following:

$$\begin{aligned} \mathrm{E}(XY) &= \sum_x \sum_y xyp(x,y) = (0)(0).1 + \cdots + (2)(1)(.4) = .9 \\ \mathrm{E}(X) &= \sum_x xp_X(x) = (0)(.3) + (1)(.3) + (2)(.4) = 1.1 \\ \mathrm{E}(Y) &= \sum_x yp_Y(x) = (0)(.3) + (1)(.7) = .7 \end{aligned}$$

Using the computational form for covariance, we found that

$$\mathrm{Cov}(X, Y) = .9 - (1.1)(.7) = .13.$$

To see how large this is, we calculate the following (**verify!**):

$$\begin{aligned} \mathrm{E}(X^2) = 1.9, &\quad \text{so} \quad \mathrm{V}(X) = 1.9 - 1.1^2 = .69, \quad \sigma_X \doteq .8307 \\ \mathrm{E}(Y^2) = .7, &\quad \text{so} \quad \mathrm{V}(Y) = .7 - .7^2 = .21, \quad \sigma_Y \doteq .4583. \end{aligned}$$

Then $\sigma_X \sigma_Y \doteq .3807$, so the natural scale for $\mathrm{Cov}(X, Y)$ is

$$-.3807 \leq \mathrm{Cov}(X, Y) \leq .3807.$$

$\mathrm{Cov}(X, Y) = .13$ is positive but small compared to .3807. To put this on an absolute scale, we compute

$$\rho = \frac{\mathrm{Cov}(X, Y)}{\sigma_X \sigma_y} = \frac{.13}{(.8307)(.4583)} \doteq .3415.$$

Since $-1 \leq \rho \leq 1$, we see that there is mild positive correlation between X and Y.