

Confidence intervals for μ

The following chart summarizes the results we have on confidence intervals for a mean μ based on a random sample of size n , say X_1, \dots, X_n from a “parent distribution” or “population” having mean μ and variance σ^2 . (The “parent distribution” is the marginal distribution of the individual X_i .)

The first two columns of the chart are assumptions on σ and the population from which the sample is taken. The third column indicates any requirement we may consequently need to place on the sample size n .

The letters z and t denote the critical values from the normal and t -tables. Specifically,

$$z = z_{\alpha/2},$$

$$t = t_{\alpha/2, n-1}.$$

The confidence level in all cases is $1 - \alpha$. In cases where the standardized sample mean has an approximately normal distribution, the confidence level is only approximate, as indicated.

The final column indicates the section in the text where each interval is discussed. (In the case given in the second row, the interval is not actually stated in the text.)

Since σ is not usually known, the third and fourth rows are the ones used in practice.

σ	Population (Parent distrib.)	n	Standardized sample mean	Confidence interval	Confidence level	Sec.
known	normal	arbitrary	$\frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$	$\bar{x} \pm \frac{z\sigma}{\sqrt{n}}$	exact	8.1
known	arbitrary	large	$\frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$	$\bar{x} \pm \frac{z\sigma}{\sqrt{n}}$	approx.	8.2
unknown	arbitrary	large	$\frac{\bar{X} - \mu}{S/\sqrt{n}}$	$\bar{x} \pm \frac{zs}{\sqrt{n}}$	approx.	8.2
unknown	normal	arbitrary	$\frac{\bar{X} - \mu}{S/\sqrt{n}}$	$\bar{x} \pm \frac{ts}{\sqrt{n}}$	exact	8.3

The table may easily be adapted for confidence bounds. Replace $\alpha/2$ by α , and use “+” or “-” depending on whether you want an upper or lower bound.

Notes:

- The table will be useful for hypothesis testing as well (Chapter 9).
- The table does *not* discuss confidence intervals for p ; see text.
- What cases does the text itself *not* discuss concerning confidence intervals for μ and p ?