MATH 731, FALL 2008
HOMEWORK SET 9
Due Wednesday, November 26 at noon

A. Find the minimal polynomial of $\sqrt{2}+\sqrt{3}$ over $\mathbb{Q}$ (and verify it is the minimal polynomial). Show that $\mathbb{Q}[\sqrt{2},\sqrt{3}] = \mathbb{Q}[\sqrt{2}+\sqrt{3}]$.

B. Let $F_n/F_{n-1}/F_{n-2}/\cdots/F_1/F_0$ be a tower of fields. Prove that F_n/F_0 is an algebraic extension if and only if F_i/F_{i-1} is an algebraic extension for each $i = 1,\ldots,n$.

C. Let E/F be an algebraic extension of fields and let $\phi : E \to E$ be an F-homomorphism. (That is, ϕ is a homomorphism of fields and $\phi(\lambda) = \lambda$ for all $\lambda \in F$.)

(1) Suppose $\alpha \in E$ and the minimal polynomial $p \in F[x]$ of α has degree n. We know $\phi^i(\alpha)$ is a root of p for all $i \in \mathbb{N}$. Show that $\phi^k(\alpha) = \alpha$ for some positive integer $k \leq n$.

(2) Conclude that ϕ is onto.