MATH 631 Midterm Exam Problem **October 18, 2001**

A version of this problem will be on the Midterm Exam next week.

In this problem, all elements and all rings are inside of $\mathbb{C}$.

Any such ring contains $\mathbb{Z}$.

(a) Let r be a nonzero algebraic integer. Prove that R is the root of a monic polynomial $f = x^n + \sum_{i=0}^{n-1} a_i x^i \in \mathbb{Z}[x]$ with the property that $a_0 \neq 0$. (The only thing new here is that we can find such an f with $a_0 \neq 0$.)

(b) Suppose that R is a ring of algebraic integers and that $r \in R$ is nonzero. Prove that there exists $s \in R$ such that rs is a positive integer.

(c) Suppose that R is a ring of algebraic integers and let

$$Q = \{\, a/b \mid a \in R, b \text{ is a positive integer} \,\}.$$

Prove that Q is a field (a subfield of $\mathbb{C}$), and hence that Q is the fraction field of R inside $\mathbb{C}$.