

## MATH 631 Homework for Chapter 1 *September 3, 1999*

Read Chapter 1 and Sections 1 & 2 of the Appendix.

The following problem is to be handed in (for a grade) on Monday, September 13. There may be additional problems assigned in a later class.

Let  $n$  and  $m$  be positive integers. Let  $A_n$  be the  $n \times n$  matrix with 1's on and above the diagonal, and zeroes below the diagonal. Thus the  $i, j$  entry of  $A_n$  is 1 if  $j \geq i$  and 0 if  $j < i$ .

For example,  $A_3 = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$ .

Compute  $(A_n)^m$  explicitly. (For example, you may give a formula or expression for the  $i, j$  entry for each  $i, j$  with  $1 \leq i, j \leq n$ .) Prove your answer is correct.

For example,  $(A_3)^2 = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}$ . If we set  $B = A_3^2$ , we can express this by saying  $b_{ij} = 0$  if  $i > j$  and  $b_{ij} = j - i + 1$  if  $j \geq i$ .

**Note:** I would like you to do two things.

- (1) Give me some idea of how you got your answer. Guessing is fine, as long as you give me an idea how you arrived at your guess.
- (2) Give a rigorous proof that your answer is correct.

For part (1), the "guessing", here is what I would really like you to do. Use the computer algebra program MAPLE to compute  $A_n^m$  for a few  $n, m$ , and try to see the pattern.

To use MAPLE, you will want to do something like the following. At the  $>$  prompt, enter  
`A := matrix([[1,1,1],[0,1,1],[0,0,1]]);`  
`evalm(A^2);`

This computes  $A_3^2$ . Some important things to notice: First, every command line ends with a `;`; and second, to set one thing equal to another, use the command `:=` (notice the `:`).

Experiment with some values of  $m, n$ , and print out a copy of the MAPLE worksheet.

One other MAPLE command you may want to use:

`binomial(a,b);` gives you the binomial coefficient  $\binom{a}{b}$ .