MATH 341 Sample Exam 1 February 21, 2002

This is a sample of what the exam on Tuesday, February 26 might look like. This is *not meant* to be exactly like the exam. You will be allowed to use a clean copy of the notes that I will hand out at the exam.

Use this sample to study by first attempting to do the problems as if you were taking the exam in class. Then, write them up well as if you were going to turn them in for a homework assignment. Both procedures will help you prepare for the exam. This sample is longer than the actual exam will be.

Each problem tells you what material from the notes you may use in solving it.

1. *In this problem you may use any material from the notes. You may use basic facts about arithmetic and the integers in this problem, but you must identify what properties you use.*
   Suppose that x is an (arbitrary) integer. Prove that if $x+x+x$ is odd, then x is odd. What type of proof did you use?

2. *In this problem you may use any material from the notes.*
   Let A, B, C be sets. Prove that if $B \cap C = \emptyset$ and $A \subseteq B$, then $A \cap C = \emptyset$.

3. *In this problem you may use any material from the notes.*
   In this problem, the variables x, y denote planets. Let $P(x)$ be the open sentence "x is between Mercury and Mars". Is the following statement true or false? Be sure to explain/justify your answer.
   $$(\exists x)\,(P(x) \text{ and } (\forall y)(P(y) \Rightarrow x = y))\,.$$

4. *In this problem you may use material from the notes up to and including Corollary 2.17.*
   Let A, B be sets. Prove that $\mathcal{P}(A \cap B) \subseteq \mathcal{P}(A) \cap \mathcal{P}(B)$.

5. *In this problem you may use any material from the notes.*
   Let A, B, C be sets. Prove or disprove the following claim:
   $$A \times (B \cap C) = (A \times B) \cap (A \times C).$$

6. *In this problem you may use any material from the notes.*
   Let A, B, C be sets. Prove or disprove the following claim:
   $$A \cup (B - C) = (A \cup B) - C.$$

7. *In this problem you may use material from the notes up to and including Theorem 2.11.*
   Let A, B, C be sets. Prove
   $$C \subseteq A \cap B \iff C \subseteq A \text{ and } C \subseteq B.$$

8. *In this problem you may use any material from the notes.*
   Let P, Q, R be statements. Are the following logical statements equivalent? **Justify your answer.**
   (a) $P \Rightarrow (Q \text{ and } R)$
   (b) $(P \Rightarrow Q)$ and $(P \Rightarrow R)$

(over)

9. *In this problem you may use any material from the notes.*
In this problem the variable x stands for a state of the U.S. Let $P(x)$ be the open sentence "There is a lake in x" and let $Q(x)$ be the open sentence "There is a mountain in x". Write *natural* English sentences for the following statements:
(a) not $(\forall x)((P(x)$ and $Q(x))$;
(b) $(\exists x)($ not $P(x)$ and $Q(x))$;
(c) not $(\exists x)(P(x)$ and $Q(x))$;
Which of these statements are true? Which are false?

10. *In this problem you may use any material from the notes.*
Suppose A, B are sets. Prove or disprove the following claim:

$$A - (A - B) = A \cap B.$$

11. *In this problem you may use any material from the notes.*
Suppose A, B, C are sets and $C \neq \emptyset$. Prove that if $A \times C = B \times C$, then $A = B$.

12. *In this problem you may use may use basic facts about arithmetic and the integers but you must identify what properties you use and you may not use facts proved in the notes.*
Suppose that x, y, z are integers and $x^2 + y^2 = z^2$.

    (a) Prove that if x, y are even, then z is even. Identify the type of proof you use.

    (b) Prove the following statement is not true: "If z is even, then x, y are even."

    (c) How would you change the statement in (b) to make it true?

13. *In this problem you may use material from the notes up to and including Definition 2.J.*
Let A, B be subsets of a set C. Prove that $A \subseteq B$ if and only if $C - B \subseteq C - A$.

14. *In this problem you may use material from the notes up to and including Theorem 2.14.*
Let A, B, C be sets. Prove that $A - (B \cup C) = (A - B) \cap (A - C)$.

15. *In this problem you may use any material from the notes.*

    (a) Give two examples of sets A with the property that $A \subseteq \mathcal{P}(A)$.

    (b) Explain why any non-empty set A whose elements are all letters of the alphabet cannot be an example of the type in part (a). (Give an explanation; do not give an example with an explicit set A.)