MATH 241 Sample Exam 1 March 6, 2001

This is a sample of what the exam on Thursday, March 8 might look like. This is *not meant* to be exactly like the exam. The exam will be "closed notes".
Use this sample to study by first attempting to do the problems as if you were taking the exam in class. Then, write them up well as if you were going to turn them in for a homework assignment. Both procedures will help you prepare for the exam. This sample is longer than the actual exam will be.

The first three problems require you to redo something covered in notes and in class – you must do this from scratch and not simply quote the corresponding results from the notes. The other problems are new; you may use results from the notes covered in class to help solve these problems.

1. Let A, B, C be sets. Prove that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

2. Let X be a set and let $f : X \to Y$, $g : Y \to Z$ be functions and assume that f is surjective. Prove (**from scratch**) that $g \circ f$ is surjective if and only if g is surjective.

3. Let A, B, C be sets. Prove that if $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$.

4. Let X be a set and let $f : X \to X$ be a function. Prove that $f \circ f$ is bijective if and only if f is bijective. (You may use the results from the notes in this problem.)

5. Suppose that x is an (arbitrary) integer. Prove that if $x + x + x$ is odd, then x is odd. What type of proof did you use?
You may use basic facts about arithmetic and the integers in this problem.

6. Prove or give a counterexample to the following statement about arbitrary sets A, B:
$$\mathcal{P}(A - B) = \mathcal{P}(A) - \mathcal{P}(B).$$

7. Let $P(x)$ be the open sentence "x lives in Wisconsin" and let $Q(x)$ be the open sentence "x drinks beer". Use $\forall, \exists,$ and , or , $\Rightarrow$, not , $P(x), Q(x)$ to express the statement "Not everybody who lives in Wisconsin drinks beer" *in symbols*, **in two different ways**.

8. Let A, B, C be sets. Prove or disprove the following claim:
$$A \times (B \cup C) = (A \times B) \cup (A \times C).$$

(over)

9. Let X be the set of all students at UWM and let Y be the set of positive integers. Define a function $f : X \to Y$ by $f(x) =$ the student number of x. (Ignore all non-numerical characters, such as dashes and spaces.)
For example, if student Nancy Zimpher has student ID 399-99-9999 55, then f(Nancy Zimpher)=39999999955.
Answer the following questions about f; be sure to include a *brief, precise* explanation of each answer.

   (a) Is f injective?

   (b) Is f surjective?

   (c) Is f bijective?

10. Let A, B, C be sets. Suppose that $A \cup C \subseteq B \cup C$ and $A \cap C \subseteq B \cap C$. Prove that $A \subseteq B$.
[Do not make any assumptions about the sets A, B, C except what you are given.]

11. Let P, Q, R be statements. Explain why the following statement is true. (You may use a truth table if you wish, or you may simply give an explanation in English.)

$$[P \Rightarrow (Q \text{ and } R)] \iff [(P \Rightarrow Q) \text{ and } (P \Rightarrow R)]$$