EE 402 Control Systems Theory — Exam 1 Review

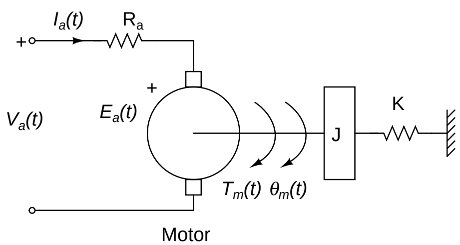
It is not necessary to turn in these review problems. They may be turned in for 1 point of extra credit per problem.

Problem 1. Homeostasis is the term used in biology for a variable regulated by a feedback mechanism. In respiration, it is actually not Oxygen demand which determines the rate of respiration, it is the blood concentration of CO2.

Draw a block diagram representing the feedback mechanism that maintains homeostasis of blood CO2.

Problem 2. For the system shown below:

1. The modeling equations are given, except for that governing motion of the load mass. Determine the modeling equation which describes motion of the load mass.
2. List the signals and parameters of the system, show the units of each.
3. Show that units balance in each of the modeling equations.
4. Draw a functional block diagram representing the system.
5. Determine the closed loop transfer function of the system.
6. Write the state space model for this system.



$$
\begin{aligned}
T_m(t) &= K_t I_a(t) \\
E_a(t) &= K_e \dot{\theta}_m(t) \\
V_{R_a}(t) &= R_a I_a(t) \\
I_a(t) &= \frac{1}{R_a}\left(V_a(t) - E_a(t)\right)
\end{aligned}
$$

Problem 3. Add PD control to the system of problem 2.

PD control:

$$u(t) = k_p e(t) + k_d \dot{e}(t) \tag{1}$$

1. Draw the new block diagram.
2. Determine the new transfer function.

Problem 4. A certain motor with a certain applied voltage has a torque-speed curve approximately given by:

$$T(t) = 100\left(e^{-\omega(t)} - e^{-3\omega(t)}\right)$$

where: $T(t)$ is torque [N-m] ω is speed [rad/sec].

An operating point of $\omega_o = 2$ [rad/sec] gives T_o = 11.7 [N-m]. This motor is connected to a load with a transfer function:

$$H(s) = \frac{b}{Js + b} \tag{2}$$

with $b = 0.1$[N-m-s], and $J = 2.0$ [N-m-s^2].

1. Determine the linearized model about the operating point for the motor.
2. Draw the block diagram for this system (Note that no external or reference input signal is defined).

Problem 5. Transfer Functions and Laplace Transforms

Provide a short answer of several sentences.

Describe the principle distinctions between a Laplace Transform and a Transfer Function.

Problem 6.

1. A. Determine y(t) corresponding to Y(s) using the graphical technique. Partial credit will be given if another technique is used.

$$Y(s) = \frac{2s + 20}{s^2 + 6s + 5}$$

2. Determine $Y(s)$ corresponding to

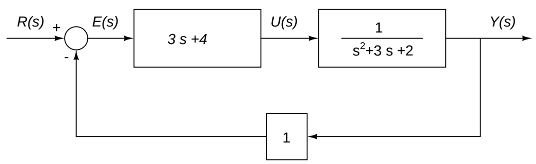
$$y(t) = 2\sin(10t)$$

Problem 7. For the functional block diagram shown below

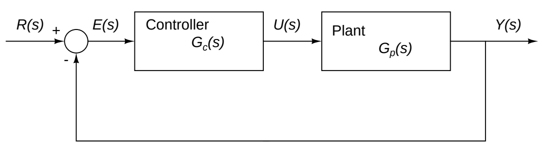
1. Determine transfer functions i and ii. Express the transfer functions in Rational Polynomial Form.

$$i)\ T(s) = \frac{Y(s)}{R(s)} \qquad ii)\ S(s) = \frac{E(s)}{R(s)}$$

2. From the block diagram, determine the time domain equations governing the system.



Problem 8. A motion control system is illustrated above.



Because of a nonlinear damper, the process dynamics are given by:

$$\dot{y}(t) + 0.1y(t)^2 = 2u(t)$$

The controller dynamics are given by:

$$\dot{u}(t) + 1.5u(t) = 3e(t)$$

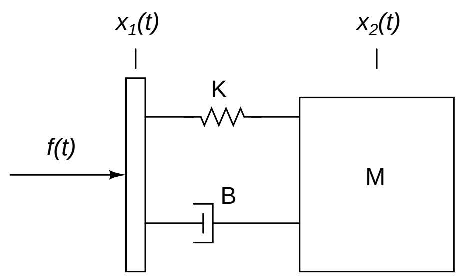
Units: $y(t)$ is in [meters] and $u(t)$ is in [volts].

1. Make a table of all signals and parameters showing units and value (if appropriate).
2. Linearize the system about the operating point $y_o = 2$[meters].
   (a) Indicate the operating point values of all signals.
3. Write the state space model for this system.
4. Write the transfer function for each individual block.

Problem 9. Determine the analogous electrical circuit

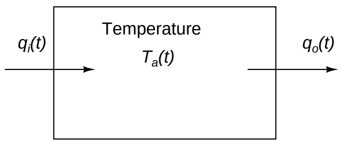
A mechanical schematic is shown below. B is a damper, K is a spring, M is a mass, $x_1(t)$ and $x_2(t)$ are positions as shown. $F(t)$ is an applied force input.

1. Determine the electrical equivalent model for this mechanical system:
   (a) Show the mechanical and electrical equivalence between all signals and parameters
   (b) show the electrical schematic
2. Expressed in terms of the mechanical parameters and in rational polynomial form:
   (a) determine the transfer function:
   $$\frac{V_1(s)}{F(s)}$$
   where $v_1(t) = \dot{x}(t)$.
   (b) determine the transfer function
   $$\frac{X_1(s)}{F(s)}$$
3. For your transfer function of part (2b), show that units balance.

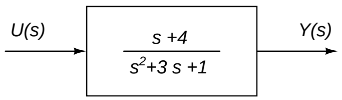


Problem 10.

1. For the thermal system shown below, with the input-output differential equation as given, determine the transfer function
$$\frac{Q_o(s)}{Q_i(s)}$$
2. For the transfer function shown below, determine the differential equation relating $y(t)$ to $u(t)$.



Thermal System



Transfer Function